as level SEQUENCES AND SERIES

Question 1

In this question you must show detailed reasoning.

$$f(x) = (1 + kx)^8$$

where k is a constant.

Given that the first 3 terms, in ascending powers of x, of the binomial series expansion of $f(x)$ are

$$1 + 8kx + px^2$$

where p is a constant,

(a) find p in terms of k.

(2)

$$g(x) = \left(a - \frac{2}{x}\right) f(x) \qquad x \neq 0$$

where a is a constant.

Given that the first 3 terms, in ascending powers of x, of the series expansion of $g(x)$ are

$$-\frac{2}{x} - 21 - 90x$$

(b) find the possible pairs of values of a and k.

(5)

Question 2

The binomial expansion of

$$(1 + ax)^{12}$$

up to and including the term in x^2 is

$$1 - \frac{15}{2}x + kx^2$$

where a and k are constants.

(a) Show that $a = -\frac{5}{8}$

(2)

(b) Hence find the value of k

(2)

Using the expansion and making your method clear,

(c) find an estimate for the value of $\left(\frac{17}{16}\right)^{12}$, giving your answer to 4 decimal places.

(2)

Question 3

. Find, in simplest form, the coefficient of x^5 in the expansion of

$$\left(5+8x^2\right)\left(3-\frac{1}{2}x\right)^6$$

(5)

Question 4

(a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(3 - \frac{2x}{9}\right)^8$$

giving each term in simplest form.

(4)

$$f(x) = \left(\frac{x-1}{2x}\right)\left(3 - \frac{2x}{9}\right)^8$$

(b) Find the coefficient of x^2 in the series expansion of $f(x)$, giving your answer as a simplified fraction.

(2)

Question 5

$$g(x) = (2 + ax)^8 \qquad \text{where } a \text{ is a constant}$$

Given that one of the terms in the binomial expansion of $g(x)$ is $3402x^5$

(a) find the value of a.

(4)

Using this value of a,

(b) find the constant term in the expansion of

$$\left(1 + \frac{1}{x^4}\right)(2 + ax)^8$$

(3)

Question 6

(a) Find the first 4 terms, in ascending powers of x, in the binomial expansion of

$$(1 + kx)^{10}$$

where k is a non-zero constant. Write each coefficient as simply as possible.

(3)

Given that in the expansion of $(1 + kx)^{10}$ the coefficient x^3 is 3 times the coefficient of x,

(b) find the possible values of k.

(3)

Question 7

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2+\frac{3x}{4}\right)^6$$

giving each term in its simplest form.

(4)

(b) Explain how you could use your expansion to estimate the value of 1.925^6
You do not need to perform the calculation.

(1)

Question 8

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{x}{16}\right)^9$$

giving each term in its simplest form.

(4)

$$f(x) = (a + bx)\left(2 - \frac{x}{16}\right)^9$$, where a and b are constants

Given that the first two terms, in ascending powers of x, in the series expansion of $f(x)$ are 128 and $36x$,

(b) find the value of a,

(2)

(c) find the value of b.

(2)