AS LEVEL Proofs

Question 1

. "$n^3 + 4n$ is prime for $n \in \mathbb{N}$" (I)

(a) Determine whether statement (I) is always true, sometimes true or never true. You must fully justify your answer.

(2)

"$n^3 + 5n$ is prime for $n \in \mathbb{N}$" (II)

(b) Determine whether statement (II) is always true, sometimes true or never true. You must fully justify your answer.

(2)

Question 2

. Prove, using algebra, that

$$n^2 + 5n$$

is even for all $n \in \mathbb{N}$

(4)

Question 3

'. In this question p and q are positive integers with $q > p$

Statement 1: $q^3 - p^3$ is never a multiple of 5

(a) Show, by means of a counter example, that Statement 1 is **not** true.

(1)

Statement 2: When p and q are consecutive **even** integers $q^3 - p^3$ is a multiple of 8

(b) Prove, using algebra, that Statement 2 is true.

(4)

Question 4

. (i) A student states

"if x^2 is greater than 9 then x must be greater than 3"

Determine whether or not this statement is true, giving a reason for your answer.

(1)

(ii) Prove that for all positive integers n,

$$n^3 + 3n^2 + 2n$$

is divisible by 6

(3)

Question 5

A student is investigating the following statement about natural numbers.

"$n^3 - n$ is a multiple of 4"

(a) Prove, using algebra, that the statement is true for all odd numbers.

(4)

(b) Use a counterexample to show that the statement is not always true.

(1)

Question 6

(a) Prove that for all positive values of a and b

$$\frac{4a}{b} + \frac{b}{a} \geqslant 4$$

(4)

(b) Prove, by counter example, that this is not true for all values of a and b.

(1)

Question 7

. Given $n \in \mathbb{N}$, prove that $n^3 + 2$ is not divisible by 8

(4)

Question 8

(i) Show that $x^2 - 8x + 17 > 0$ for all real values of x

(3)

(ii) "If I add 3 to a number and square the sum, the result is greater than the square of the original number."

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)