ALGEBRAIC MANIPULATION

SOLUTIONS - GCSE

Q1

Show that $\frac{2}{3x} + \frac{8x-6y}{12xy} - \frac{y-14x}{6xy}$ simplifies to $\frac{k}{y}$ where k is an integer.

multiply each term by LCM of denominators

$$(12xy)\frac{2}{3x} + \frac{8x-6y}{12xy}(12xy) - \frac{(y-14x)}{6xy}12xy$$

$$= \frac{8y + 8x - 6y - 2y + 28x}{12xy} \checkmark \ 1$$

$$= \frac{36x}{12xy} \quad 1$$

$$= \frac{4}{y} \quad 1$$

(3 marks)

QUESTION 2

Make p the subject of the formula $t = \frac{4-3p}{3(3p-5)}$

$$t = \frac{4-3p}{9p-15}$$

$$9pt - 15t = 4 - 3p \quad 1$$

$$9pt + 3p = 15t + 4 \quad 1$$

$$p(9t+3) = 15t+4 \quad 1$$

$$p = \frac{15t+4}{9t+3} \quad 1$$

(4 marks)

QUESTION 3

Expand and simplify $(4x - 2)(3x + 1)(x - 7)$

$(4x-2)(3x^2-20x-7)$ (1)

$= 12x^3 - 80x^2 - 28x - 6x^2 + 40x + 14$ (1)

$= 12x^3 - 86x^2 + 12x + 14$

$12x^3 - 86x^2 + 12x + 14$ (1)

(3 marks)

QUESTION 4

$2^x = \dfrac{2^n}{\sqrt[5]{2}}$ $\quad 2^y = (\sqrt[3]{2})^7$

Given that $x + y = 6$

work out the value of n.

$2^x = \dfrac{2^n}{2^{1/5}}$

$2^x = 2^{n - \frac{1}{5}}$

$x = n - \frac{1}{5}$ (1)

$2^y = 2^{7/3}$

$y = \frac{7}{3}$ (1)

If $x + y = 6$

$n - \frac{1}{5} + \frac{7}{3} = 6$

$n + \frac{32}{15} = 6$

$n = \frac{58}{15}$

$n =$ $3\frac{13}{15}$ (1)

(3 marks)

QUESTION 5

The curve **C** has equation $y = 2x^2 - 20x + 30$

Find the coordinates of the turning point on **C**.

$y = 2(x^2 - 10x + 15)$

$y = (x-5)^2 - 25 + 15$ (1)

$y = (x-5)^2 - 10$ (1)

Examiner Tip

Turning point = Complete the square

(5 , −10) (1)

(3 marks)

Question 6

(a) Simplify fully $\dfrac{(b+9)^2}{7(b+9)}$ $= \dfrac{(b+9)\cancel{(b+9)}}{7\cancel{(b+9)}}$

$\dfrac{b+9}{7}$ (1)

(1)

(b) Factorise $\overset{-12}{3k^2 - k - 4}$

$\dfrac{(3k-4)(3k+3)}{3}$ (1) =

$= (3k-4)(k+1)$ (1)

(2)

(c) Simplify fully $\dfrac{x+3}{x+4} \div \dfrac{9-x^2}{x^2+4x}$

$$\frac{x+3}{x+4} \div \frac{(3-x)(3+x)}{x(x+4)} \quad (1)$$

$$= \frac{\cancel{x+3}}{\cancel{x+4}} \times \frac{x\cancel{(x+4)}}{(3-x)\cancel{(3+x)}} \quad (1)$$

$$= \frac{x}{3-x} \quad (1)$$

(3)

Question 7

Write $\frac{7}{9x-21} + \left[(x+2) \div \frac{3x^2-x-14}{3x+2}\right]$ in the form $\frac{ax+b}{cx+d}$ where a and b are integers.

$$= \frac{7}{9x-21} + \left[\frac{x+2}{1} \times \frac{3x+2}{3x^2-x-14}\right]$$

$3x^2 - x - 14$

$(3x-7)\frac{(3x+6)}{3}$

$\Rightarrow (3x-7)(x+2)$

$$= \frac{7}{9x-21} + \left[\frac{x+2}{1} \times \frac{3x+2}{(3x-7)(x+2)}\right] \quad (1)$$

$$= \frac{7}{9x-21} + \frac{3x+2}{3x-7}$$

$$= \frac{7}{3(3x-7)} + \frac{3x+2}{3x-7} \quad (1)$$

$$= \frac{7 + 3(3x+2)}{3(3x-7)} \quad (1)$$

$$= \frac{7+9x+6}{3(3x-7)} = \frac{9x+13}{9x-21} \quad (1)$$

(4)

Question 8

Make m the subject of $h = \frac{5m}{8} - p$

$8h = 5m - 8p$ (1)

$8h + 8p = 5m$ (1)

$m = \frac{8h + 8p}{5}$ (1)

m=......................(3)

Question 9

(i) Write $x^2 - 10x + 8$ in the form $(x - a)^2 - b$ where a and b are integers.

$(x-5)^2 - 25 + 8$ (1)

$(x-5)^2 - 17$ (1)

(2)

(ii) Hence, write down the coordinates of the turning point on the graph of $y = x^2 - 10x + 8$

(5, −17) (1)

(1)

Question 10

Write $\frac{(10x^6 y^4)^2}{4x^2 y^5 \times 5xy^{-4}}$ in the form $ax^b y^c$ where a, b and c are integers.

$= \frac{100x^{12}y^8}{20x^3 y}$ (1) (1)

$5x^9 y^7$ (1)

(3)

Question 11

Make x the subject of the formula $y = \dfrac{5(3x-2)}{7x+4}$

$7xy + 4y = 15x - 10$ (1)

$4y + 10 = 15x - 7xy$ (1)

$4y + 10 = x(15 - 7y)$ (1)

$x = \dfrac{4y+10}{15-7y}$ (1) **(4)**

Question 12

Show that $\dfrac{x^2 - x - 12}{2x^2 + 5x - 3}$ can be written in the form $\dfrac{ax+b}{cx+d}$ where a, b, c and d are integers.

$x^2 - x - 12 = (x-4)(x+3)$ (1)

-6

$2x^2 + 5x - 3 = \dfrac{(2x+6)(2x-1)}{2}$

$= (x+3)(2x-1)$ (1)

$\therefore \dfrac{x^2 - x - 12}{2x^2 + 5x - 3} = \dfrac{(x-4)\cancel{(x+3)}}{(2x-1)\cancel{(x+3)}}$ (1)

$= \dfrac{x-4}{2x-1}$ (1)

(4)

Question 13

Show algebraically that the sum of any 3 consecutive even numbers is always a multiple of 6.

Let the even numbers be $2n$, $2n+2$, $2n+4$

Sum: $2n + 2n + 2 + 2n + 4$ (1)

$= 6n + 6$ (1)

$= 6(n+1)$ ∴ multiple of 6. (1)

(3)

Question 14

n is a whole number.

(a) Prove that $n^2 + (n + 1)^2$ is always an odd number.

$= n^2 + n^2 + 2n + 1$

$= 2n^2 + 2n + 1$ (1)

$= 2(n^2 + n) + 1$ (1)

$\Rightarrow$ odd number

(2)

n and a are integers.

Cynthia says that $(n^2 - a^2) - (n - a)^2$ is always an integer.

(b) Is Cynthia correct?
You must give reasons for your answer.

$n^2 - a^2 - (n^2 - 2an + a^2)$

$= n^2 - a^2 - n^2 + 2an - a^2$

$= 2an - 2a^2$

$= 2a(n - a)$ (1)

a and n are integers
so $n - a$ is an integer (1)
and product of integers is an integer
hence $2a(n-a)$ is an integer.

(2)

Question 15

$\frac{2x+8}{x-6} + \frac{2x+2}{x+6} - 4$ can be written in the form $\frac{ax+b}{x^2-36}$ where a and b are integers.

Work out the value of a and the value of b.
You must show all your working.

$$\left[\frac{2x+8}{x-6} + \frac{2x+2}{x+6}\right] - 4$$

$$= \left[\frac{(x+6)(2x+8) + (x-6)(2x+2)}{(x-6)(x+6)}\right] - 4$$

$$= \left[\frac{2x^2+20x+48+2x^2-10x-12}{(x-6)(x+6)}\right] - 4 \quad 1$$

$$= \frac{4x^2+10x+36}{(x-6)(x+6)} - \frac{4}{1} \quad 1$$

$$\frac{4x^2+10x+36 - (4x^2-144)}{x^2-36}$$

$$\rightarrow \frac{10x+180}{x^2-36} \quad 1$$

$a =$ 10

$b =$ 180

(3 marks)

Question 16

(a) Factorise fully $6m^2 - 54$

$6(m^2 - 9)$

1 1

$6(m-3)(m+3)$

(2)

(*b*) Show that $(p + 6)(3p - 2)(2p + 3)$ can be written in the form $ap^3 + bp^2 + cp + d$ where a, b, c and d are integers

$(p+6)(6p^2+5p-6)$ 1

$= 6p^3 + 5p^2 - 6p + 36p^2 + 30p - 36$ 1

$= \underline{6p^3 + 41p + 24p - 36}$ 1

(3)

Question 17

Show that $\dfrac{4x}{x+3} - \dfrac{2x+1}{x-3} - 2$ can be written in the form $\dfrac{ax+b}{x^2-9}$ where a and b are integers.

$$\frac{4x}{x+3} - \frac{2x+1}{x-3} - \frac{2}{1}$$

$$= \frac{4x(x-3) - (x+3)(2x+1) - 2(x^2-9)}{(x+3)(x-3)} \quad 1$$

$$= \frac{4x^2 - 12x - (2x^2 + 7x + 3) - 2(x^2 - 9)}{x^2 - 9} \quad 1$$

$$= \frac{4x^2 - 12x - 2x^2 - 7x - 3 - 2x^2 + 18}{x^2 - 9} \quad 1$$

$$= \frac{-19x + 15}{x^2 - 9} \quad 1$$

..................................**(4)**

Question 18

Show that $\frac{8x^2}{(4x^2-64)} \div \frac{4x^3}{2(x-4)}$ can be written in the form $\frac{1}{x(x+r)}$ where r is an integer.

$$\frac{8x^2}{4(x^2-16)} \div \frac{4x^3}{2(x-4)} \quad 1$$

$$= \frac{\cancel{8}x^{\cancel{2}}}{\cancel{4}(\cancel{x-4})(x+4)} \times \frac{\cancel{2}(\cancel{x-4})}{\cancel{4x^3}} \quad 1$$

$$= \frac{1}{x(x+4)} \quad 1$$

(3 marks)

Question 19

-Make d the subject of the formula $c = \frac{5(2-d)}{d+3}$

$$c = \frac{10-5d}{d+3} \quad 1$$

$$cd + 3c = 10 - 5d$$

$$cd + 5d = 10 - 3c \quad 1$$

$$d(c+5) = 10 - 3c \quad 1$$

$$d = \frac{10-3c}{c+5} \quad 1$$

(4)

Question 20

Express $\frac{3x}{x+1} + \frac{6x}{x-2}$ as a single fraction in its simplest form.

$$= \frac{3x(x-2) + 6x(x+1)}{(x+1)(x-2)} \quad 1$$

(3)

$$= \frac{3x^2 - 6x + 6x^2 + 6x}{(x+1)(x-2)} \quad 1 \quad = \frac{9x^2}{(x+1)(x-2)} \quad 1$$

Question 21

Show that $\dfrac{9x-18}{(x+5)(x-2)} \div \dfrac{x-5}{x^3-25x}$ simplifies to ax where a is an integer.

$$= \frac{9(x-2)}{(x+5)(x-2)} \div \frac{x-5}{x(x^2-25)} \quad 1$$

$$= \frac{9\cancel{(x-2)}}{\cancel{(x+5)}\cancel{(x-2)}} \times \frac{x\cancel{(x-5)}\cancel{(x+5)}}{\cancel{x-5}} \quad 1 \quad 1$$

$$= 9x \quad 1$$

(4)