ELIJAH MATHS
GCSE and A Level Maths

VECTORS

Solutions

Question 1

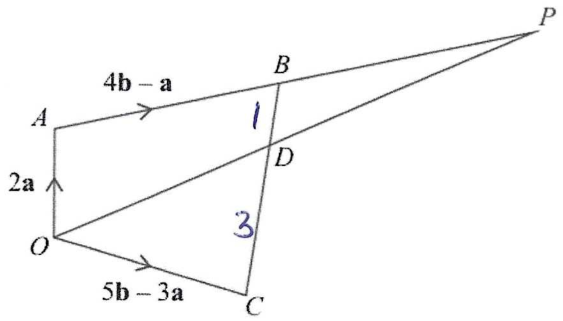

Diagram **NOT** accurately drawn

OABC is a quadrilateral.
ABP and *ODP* are straight lines.

$\overrightarrow{OA} = 2\mathbf{a}$ $\quad \overrightarrow{AB} = 4\mathbf{b} - \mathbf{a}$ $\quad \overrightarrow{OC} = 5\mathbf{b} - 3\mathbf{a}$

(a) Find an expression in terms of **a** and **b** for the vector $\overrightarrow{BC}$
Simplify your answer.

$BC = BA + AO + OC$

$= a - 4b - 2a + 5b - 3a$ (1)

$b - 4a$ (1) ....**(2)**

The point *D* lies on *BC* such that $BD : DC = 1 : 3$

Given that $\overrightarrow{OP} = n\overrightarrow{OD}$

(b) use a vector method to find the value of *n*

$\overrightarrow{BD} = \frac{1}{4} BC$

$= \frac{1}{4}(b - 4a)$

$= \frac{1}{4} b - a$ (1)

$\overrightarrow{OD} = 2a + 4b - a + \frac{1}{4} b - a$

$= \frac{17}{4} b.$

$\overrightarrow{OP} = \frac{17}{4} n b$ (1)

$\overrightarrow{OP} = 2a + 4b - a + k(4b - a)$

or $\overrightarrow{OP} = 2a + k(4b - a)$

$OP = 2a + 4kb - ak.$

$\overrightarrow{OP} = (2 - k)a + 4kb.$ (1)

Compare co-efficients

$\frac{17}{4} n = 4k$ and $2 - k = 0$

$k = 2$

$\frac{17}{4} n = 8$

....................................**(4)**

$\boxed{n = \frac{32}{17}}$ (1)

Question 2

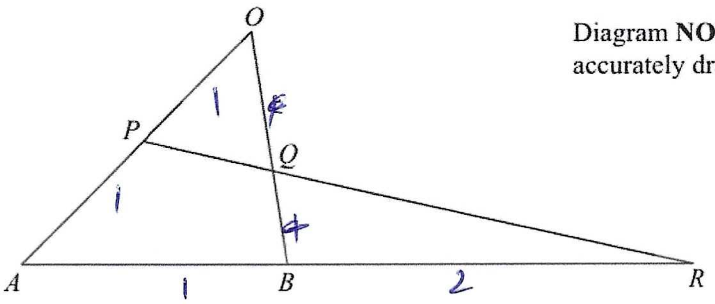

Diagram **NOT** accurately drawn

OAB is a triangle.
P is the midpoint of *OA*
Q is a point on *OB*

ABR and *PQR* are straight lines.

$\overrightarrow{OA} = 12\mathbf{a}$ $\qquad \overrightarrow{OB} = 8\mathbf{b}$

(a) Express $\overrightarrow{AB}$ in terms of **a** and **b**

$\overrightarrow{AB} = -OA + OB$
$= 8b - 12a$

$8b - 12a$ (1)

(1)

$AB : BR = 1 : 2 \qquad \overrightarrow{OQ} = n\mathbf{b}$

(b) Use a vector method to find the value of n

$OQ = nb$
$PR = 6a + 3(8b - 12a)$
$= 6a + 24b - 36a$
$= 24b - 30a.$ (1)

$\overrightarrow{OR} = nb + m(24b - 30a)$
$= nb + 24mb - 30ma$
$= (n + 24m)b - 30ma.$ (1) (1)

$\overrightarrow{OR} = 12a + 3(8b - 12a)$
$= 12a + 24b - 36a$
$= 24b - 24a$

$-30m = -24$
$m = \frac{4}{5}$ (1)

$n + 24m = 24$
$n + \frac{96}{5} = 24$

$n =$ $n = \frac{24}{5}$ (1)

(4)

Question 3

OAB is a triangle.

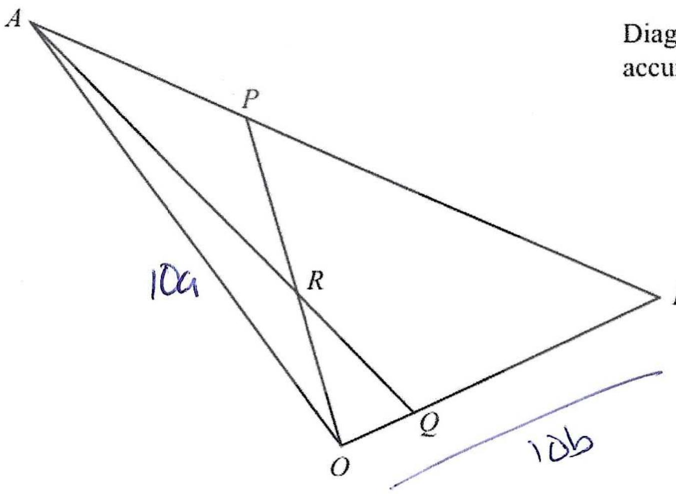

Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 10\mathbf{a}$ $\quad \overrightarrow{OB} = 10\mathbf{b}$

ARQ and ORP are straight lines.

$\overrightarrow{AP} = \frac{1}{4}\overrightarrow{AB}$ and $\overrightarrow{OQ} = \frac{1}{5}\overrightarrow{OB}$

$AP = \frac{1}{4}(10b - 10a) = \frac{5}{2}b - \frac{5}{2}a$

$OQ = \frac{1}{5} \times 10b = 2b$

Write the following vectors in terms of **a** and **b**
Simplify your answers.

(i) $\overrightarrow{AQ}$ $= -\overrightarrow{OA} + \overrightarrow{OQ}$

$= -10a + 2b$ 1

....
(1)

(ii) $\overrightarrow{OP}$ $= \overrightarrow{OA} + \overrightarrow{AP}$

$= 10a + \frac{5}{2}b - \frac{5}{2}a$

$= \frac{15}{2}a + \frac{5}{2}b$ 1

....
(1)

$= nOP$

$= n\left(\frac{15}{2}a + \frac{5}{2}b\right)$

$= \frac{15}{2}na + \frac{5}{2}nb$ (1)

(iii) $\overrightarrow{OR}$

$OR = OA + kAB$

$= 10a + k(-10a + 2b)$

$= 10a - 10ka + 2kb$

$= (10 - 10k)a + 2kb$ (1)

Compare co-efficients

$\frac{15}{2}n = 10 - 10k$

$15n = 20 - 20k$

$15n + 20k = 20 \quad (i)$

$\frac{5}{2}n = 2k$

$5n = 4k$

$5n - 4k = 0 \quad (ii)$

Solve simultaneously (i) and (ii)

$15n + 20k = 20$

$5n - 4k = 0) \times 5$ (1)

$15n + 20k = 20$

$+(25n - 20 = 0$

$40n = 20$

$n = \frac{1}{2}, \quad k = \frac{5}{8}$

$OR = \frac{15}{2} \times \frac{1}{2}a + \frac{5}{2} \times \frac{1}{2}b$

$OR = \frac{15}{4}a + \frac{5}{4}b$ (1)

(4)

Question 4

Here are two vectors.

$\overrightarrow{FG} = \begin{pmatrix} -5 \\ 2 \end{pmatrix} \qquad \overrightarrow{HG} = \begin{pmatrix} 4 \\ 14 \end{pmatrix}$

Calculate the magnitude of the vector $\overrightarrow{HF}$

$HF = HG + GF$

$= \begin{pmatrix} 4 \\ 14 \end{pmatrix} + \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} 9 \\ 12 \end{pmatrix}$ (1)

$|\overrightarrow{HF}| = \sqrt{9^2 + 12^2}$ (1)

15 **(3)** (1)

Question 5

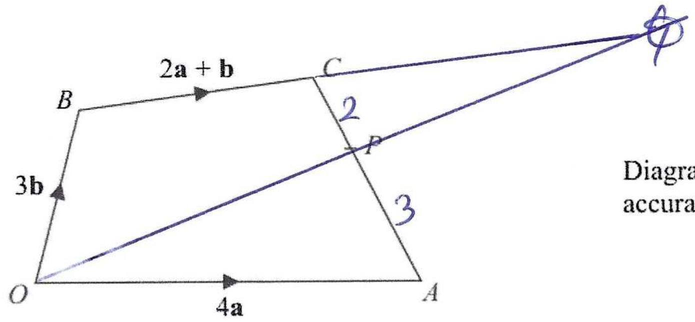

Diagram **NOT** accurately drawn

The diagram shows a quadrilateral *OACB* in which

$\overrightarrow{OA} = 4\mathbf{a}$ $\quad \overrightarrow{OB} = 3\mathbf{b}$ $\quad \overrightarrow{BC} = 2\mathbf{a} + \mathbf{b}$

(a) Find $\overrightarrow{AC}$ in terms of **a** and **b**
Give your answer in its simplest form.

$AC = -4a + 3b + 2a + b$ 1

$= -2a + 4b$

$\overrightarrow{AC} =$ $-2a + 4b$ 1
(2)

The point *P* lies on *AC* such that $AP : PC = 3 : 2$

The point *Q* is such that *OPQ* and *BCQ* are straight lines.

(b) Using a vector method, find $\overrightarrow{OQ}$ in terms of **a** and **b**
Give your answer in its simplest form.
Show your working clearly.

$O\cancel{P}Q = n\overrightarrow{OP} = n\left[4a + \frac{3}{5}(-2a + 4b)\right]$

$= \frac{14}{5}na + \frac{12}{5}nb$ 1

$OQ = 3b + k(2a + b)$

$= 2ka + (k+3)b$ 1

$2k = \frac{14}{5}n$

$10k = 14n$

$5k = 7n$

$k + 3 = \frac{12}{5}n$ 1

$5k + 15 = 12n$

$7n + 15 = 12n$

$5n = 15$

$n = 3$ $\rightarrow OQ =$

$\overrightarrow{OQ} =$ $\frac{42}{5}a + \frac{36}{5}b$ 1
(4)

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Question 6

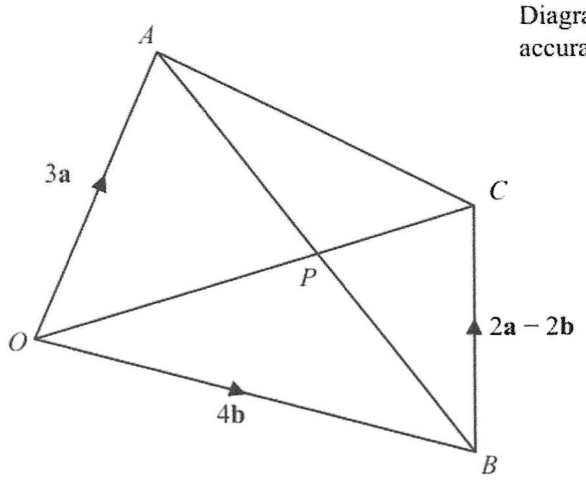

Diagram **NOT** accurately drawn

OACB is a quadrilateral.

$\overrightarrow{OA} = 3\mathbf{a}$ $\quad\overrightarrow{OB} = 4\mathbf{b}$ $\quad\overrightarrow{BC} = 2\mathbf{a} - 2\mathbf{b}$

(a) (i) Find the vector $\overrightarrow{OC}$ in terms of **a** and **b**
Simplify your answer.

$OC = 4b + 2a - 2b$

$\overrightarrow{OC} =$ $2a + 2b$ (1)

(1)

(ii) Find the vector $\overrightarrow{AB}$ in terms of **a** and **b**

$\overrightarrow{AB} =$ $4b - 3a$ (1)

(1)

The point *P* lies on *AB* and on *OC*

(b) Using a vector method, find the ratio *AP* : *PB*
Show your working clearly.

$AP = n\,\overrightarrow{AB}$
$= n(4b - 3a)$
$= 4nb - 3na$

$AP = -3a + k(2a + 2b)$ (1)
$= -3a + 2ka + 2kb$
$= (2k - 3)a + 2kb$

Compare co-efficients

$4n = 2k$ and $-3n = 2k - 3$
$3 - 3n = 2k$ (1)

$4n = 3 - 3n$
$7n = 3$
$n = \frac{3}{7}$

$AP = \frac{3}{7}\overrightarrow{AB}$

(3)

$AP : PB = 3 : 4$ (1)

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Question 7

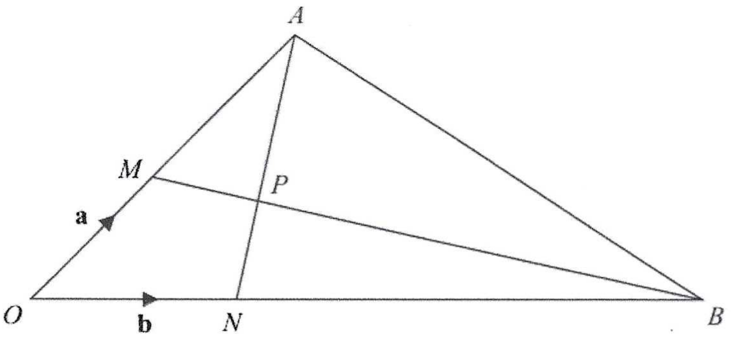


Diagram **NOT** accurately drawn

OMA, *ONB*, *MPB* and *NPA* are straight lines.

M is the midpoint of *OA*

$ON:NB = 1:5$

$\overrightarrow{OM} = \mathbf{a}$ $\overrightarrow{ON} = \mathbf{b}$

(a) Find in terms of **a** and **b** the vector $\overrightarrow{AN}$

$-2a + b$ (1)

(1)

(b) Use a vector method to find the ratio $AP:PN$

$AP = nAN$
$= n(b-2a)$
$= nb - 2na$

$AP = \overrightarrow{AM} + k\overrightarrow{MB}$ (1)
$= -a + k(-a+6b)$
$= -a - ka + 6kb$
$= (-1-k)a + 6kb$

Compare co-efficients

$n = 6k$ and $-2n = -1-k$ (1)

$-2(6k) = -1-k$
$-12k = -1-k$
$11k = 1 \Rightarrow k = \frac{1}{11}$ (1)

$n = \frac{6}{11}$

$\overrightarrow{AP} = \frac{6}{11}\overrightarrow{AN}$

$AP:PN =$ 6 : 5 (1)

(4)

Question 8

OAED is a quadrilateral.

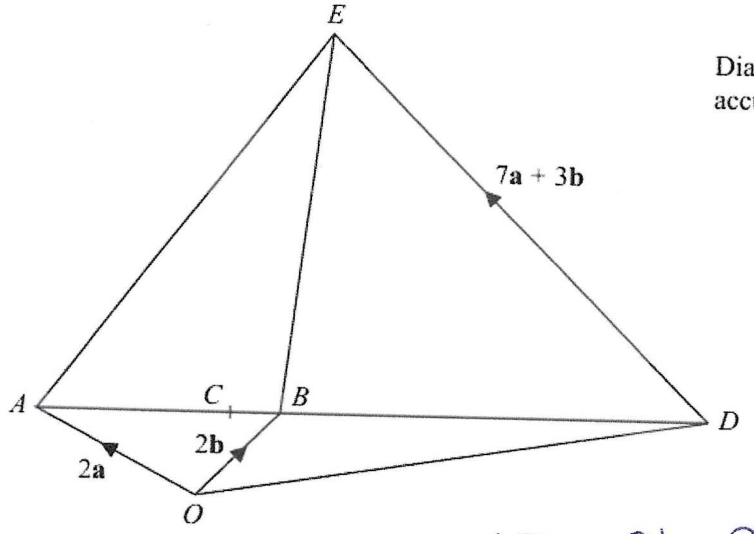


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 2\mathbf{a}$ $\overrightarrow{OB} = 2\mathbf{b}$ $\overrightarrow{DE} = 7\mathbf{a} + 3\mathbf{b}$

$AB : BD = 1 : 2$

The point C on AB is such that OCE is a straight line.

Use a vector method to find the ratio of $OC : CE$

$AB = 2b - 2a$ 1

$AD = 3(2b - 2a) = 6b - 6a$

$OE = 2a + 6b - 6a + 7a + 3b$

$= 3a + 9b$ 1

$\overrightarrow{OC} = n\,\overrightarrow{OE}$

$= 3na + 9nb$ 1

and

$\overrightarrow{OC} = 2a + k(2b - 2a)$

$= 2a + 2kb - 2ka$ 1

$= (2 - 2k)a + 2kb$

Compare co-efficients

$3n = 2 - 2k$ and $9n = 2k$ 1

$3n = 2 - 9n$

$12n = 2$

$n = \frac{1}{6}$

$\therefore \overrightarrow{OC} = \frac{1}{6} OE$

$OC : CE = 1 : 5$ 1

(6)

Question 9

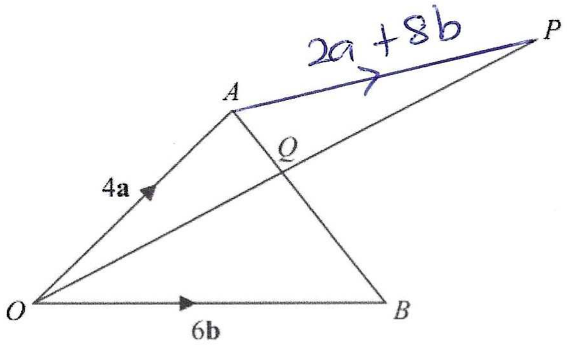


Diagram **NOT** accurately drawn

OAB is a triangle.
Q is the point on *AB* such that *OQP* is a straight line.

$\overrightarrow{OA} = 4\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$ $\overrightarrow{AP} = 2\mathbf{a} + 8\mathbf{b}$

Using a vector method, find the ratio $AQ : QB$

$AB = 6b - 4a$
$OP = 6a + 8b$ 1

$AQ = nAB$
$= n(6b - 4a)$
$= 6nb - 4na.$ 1

$AQ = -4a + m(OP)$
$= -4a + m(6a + 8b)$
$= -4a + 6ma + 8mb$
$= (6m - 4)a + 8mb$ 1

Compare co-efficients

$6n = 8m$ and $\underline{-4n = 6m - 4}$
2

$= (-2n = 3m - 2) \times 3$
$= -6n = 9m - 6$ 1

$-8m = 9m - 6$
$6 = 17m$
$m = \frac{6}{17}$

$6n = \frac{48}{17}$
$n = \frac{48}{102}$
$= \frac{8}{17}$ 1

$AQ : QB =$ 8 : 9. 1 **(5)**

? **Question 10**

ABCDEF and *GHIJKL* are regular hexagons each with centre *O*.

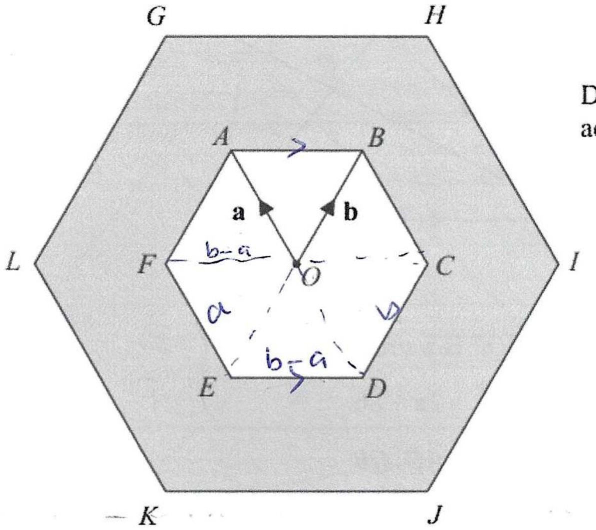


Diagram **NOT** accurately drawn

GHIJKL is an enlargement of *ABCDEF*, with centre *O* and scale factor 2

$\overrightarrow{OA} = \mathbf{a}$ $\quad \overrightarrow{OB} = \mathbf{b}$

(a) Write the following vectors, in terms of **a** and **b**.
Simplify your answers.

(i) $\overrightarrow{AB}$

$b - a$ (1)

.... (1)

(ii) $\overrightarrow{KI}$

$2(b-a) + 2b$

$2b - 2a + 2b$ (1)

$4b - 2a.$ (1)

.... (2)

(iii) $\overrightarrow{LD}$ $\quad 2(b-a) - a$

$2b - 2a - a$ (1)

$2b - 3a.$ (1)

.... (2)

The triangle OAB has an area of $5\,\text{cm}^2$

(b) Calculate the area of the shaded region.

Unshaded $= 5 \times 6 = 30\,\text{cm}^2$ (1)

Area of big hexagon $= 30 \times 4$

$= 120\,\text{cm}^2$ (1)

Scale factor $= 1:2$

Area factor $= 1:4$

Shaded $= 120 - 30$

$= 90\,\text{cm}^2$ (1)

.... cm^2

(3)

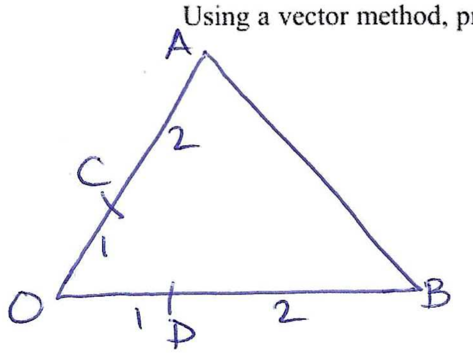
Question11

OAB is a triangle.

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

The point C lies on OA such that $OC : CA = 1 : 2$
The point D lies on OB such that $OD : DB = 1 : 2$

Using a vector method, prove that $ABDC$ is a trapezium.



$\overrightarrow{AB} = b - a$ (1)

$\overrightarrow{CD} = \frac{1}{3}b - \frac{1}{3}a$

$\overrightarrow{CD} = \frac{1}{3}(b - a)$ (1)

$CD = \frac{1}{3}AB$

hence AB is paallel to CD

So ABDC is a trapezium (1)

........................**(3)**

Question 12

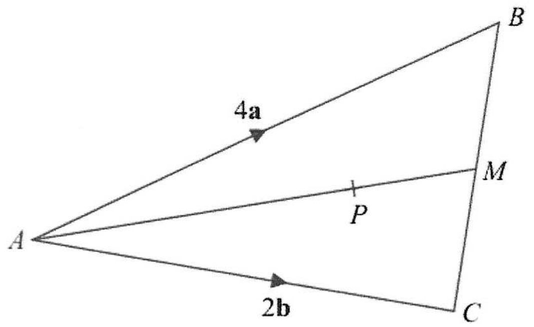


Diagram **NOT** accurately drawn

ABC is a triangle.
The midpoint of *BC* is *M*.
P is a point on *AM*.

$\overrightarrow{AB} = 4\mathbf{a}$

$\overrightarrow{AC} = 2\mathbf{b}$

$\overrightarrow{AP} = \frac{3}{2}\mathbf{a} + \frac{3}{4}\mathbf{b}$

Find the ratio $AP : PM$

$AM = n\left(\frac{3}{2}a + \frac{3}{4}b\right) = \frac{3}{2}na + \frac{3}{4}nb$ (1)

$AM = 4a + \frac{1}{2}(2b - 4a) = 2a + b$ (1)

Compare Co-efficients

$\frac{3}{2}n = 2$

$n = \frac{4}{3}$ (1)

$AM = \frac{4}{3}AP$

$3AM = 4AP$

$AP = \frac{3}{4}AM$

$AP : PM = 3 : 1$ ....**(4)** (1)

Question 12

Here are two vectors.

$$\overrightarrow{AB} = \begin{pmatrix} 6 \\ -9 \end{pmatrix} \qquad \overrightarrow{CB} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Find the magnitude of $\overrightarrow{AC}$.

$$AC = AB + BC$$
$$= \begin{pmatrix} 6 \\ -9 \end{pmatrix} + \begin{pmatrix} -1 \\ -3 \end{pmatrix} \quad 1$$
$$= \begin{pmatrix} 5 \\ -12 \end{pmatrix} \quad 1 \quad 1$$

........................(3)

$$|\overrightarrow{AC}| = \sqrt{5^2 + (-12)^2}$$
$$= 13$$