ELIJAH MATHS GCSE and A Level Maths

FUNCTIONS

You must write down all the stages in your working.

Q1

g is the function with domain $x \geqslant -3$ such that $g(x) = x^2 + 6x$

(a) Write down the range of g^{-1}

$f^{-1}(x) \geqslant -3$

(1)

(b) Express the inverse function g^{-1} in the form $g^{-1} : x \mapsto \ldots$

$y = x^2 + 6x$

$y = (x+3)^2 - 9$

$\sqrt{y+9} = x+3$

$x = -3 + \sqrt{x+9}$

$g^{-1}(x) = -3 + \sqrt{x+9}$

$g^{-1} : x \mapsto$ ....

(4)

Q2

The functions f and g are such that

$$f(x) = x^2 - 2x \qquad g(x) = x + 3$$

The function h is such that $h(x) = fg(x)$ for $x \geqslant -2$

Express the inverse function $h^{-1}(x)$ in the form $h^{-1}(x) = \ldots$

$h(x) = f(x+3) = (x+3)^2 - 2(x+3)$

$= x^2 + 6x + 9 - 2x - 6$

$= x^2 + 4x + 3$

complete the square

$h(x) = (x+2)^2 - 4 + 3$

$y = (x+2)^2 - 1$

Make x subject of formula

$(x+2)^2 = y+1$

$x = -2 + \sqrt{y+1}$

$h^{-1}(x) = -2 + \sqrt{x+1}$

[5]

Q3

The function f is such that $f(x) = x^2 - 8x + 5$ where $x \leqslant 4$

Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$

Complete the Square

$y = (x-4)^2 - 16 + 5$

$y = (x-4)^2 - 11$

Make x subject of formula

$(x-4)^2 = y + 11$

$x - 4 = \sqrt{y+11}$

$x = 4 + \sqrt{y+11}$

$f^{-1}(x) =$ $4 + \sqrt{x+11}$ **(3)**

Q4

The function f is such that $f(x) = \dfrac{k}{x}$ where $x \neq 0$ and k is an integer.

(a) Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$

let $y = \dfrac{k}{x}$

$xy = k$

$x = \dfrac{k}{y}$

$f^{-1}(x) =$ $\dfrac{k}{x}$

(1)

The function g is such that $g(x) = 2 - 3x^4$ where $x \neq 0$

The function h is such that $h(x) = \dfrac{3x}{2 - x}$ where $x \neq 2$

(b) (i) Find g(−2)

$$g(-2) = 2 - 3(-2)^4$$
$$= 2 - 3(16)$$

-46

(1)

(ii) Express the composite function hg in the form hg(x) = ...
Give your answer in its simplest form.

$$hg(x) = h(2 - 3x^4)$$
$$= \frac{3(2 - 3x^4)}{2 - (2 - 3x^4)}$$
$$= \frac{6 - 9x^4}{3x^4}$$

$hg(x) = \dfrac{6 - 9x^4}{3x^4}$

(2)

Q5

$f(x) = 9 - \sqrt{x}$ where $x \geqslant 0$
$g(x) = 4x^2$

(a) Find f(9)

$$f(9) = 9 - \sqrt{9}$$
$$= 9 - 3$$
$$= 6$$

6

(1)

(b) Solve $\mathrm{fg}(x) < 0$
Show clear algebraic working.

$f(4x^2) < 0$

$9 - \sqrt{4x^2} < 0$

$9 - 2x < 0$

$9 < 2x$

$4\frac{1}{2} < x$

$x > 4\frac{1}{2}$

(3)

Q6

The function f is defined as

$$\mathrm{f}: x \mapsto \frac{3x+1}{x-2}$$

(a) State the value of x that cannot be included in any domain of the function f

$x \neq 2$

(1)

(b) Express the inverse function f^{-1} in the form $\mathrm{f}^{-1}(x) = \ldots$

$y = \frac{3x+1}{x-2}$

(3)

$xy - 2y = 3x + 1$

$xy - 3x = 2y + 1$

$x(y-3) = 2y + 1$

$x = \frac{2y+1}{y-3}$

$f^{-1}(x) = \frac{2x+1}{x-3}$

Q7

The functions f and g are such that

$$f : x \mapsto 5x + 7$$
$$g : x \mapsto \frac{5}{2x - 9}$$

(a) State which value of x cannot be included in any domain of g

$x \neq 4.5$

(1)

(b) Find fg(4)

$f\left(\frac{5}{8-9}\right) = f(-5)$

$= -25 + 7$

-18

(2)

The function h is such that

$$h : x \mapsto 3x^2 - 12x + 8 \quad \text{where } x > 2$$

(c) Express the inverse function h^{-1} in the form $h^{-1} : x \mapsto ...$

$y = 3[x^2 - 4x] + 8$

$= 3[(x-2)^2 - 4] + 8$

$= 3(x-2)^2 - 12 + 8$

$y = 3(x-2)^2 - 4$

$(x-2) = \sqrt{\frac{y+4}{3}}$

$x = 2 + \sqrt{\frac{y+4}{3}}$

$h^{-1}(x) = 2 + \sqrt{\frac{x+4}{3}}$

(4)

Q8

The functions f and g are such that

$$f(x) = 3x - 2 \qquad\qquad g(x) = \frac{x}{2x-1}$$

(a) Find g(3)

$g(3) = 9 - 2$

7

....
(1)

(b) Find gf(x)
Give your answer in its simplest form.

$$g(3x-2) = \frac{3x-2}{6x-4-1}$$

$$gf(x) = \frac{3x-2}{6x-5}$$

gf(x) = ...
(2)